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Determination of the amount of entropy of non-recoverable elements of the technical system

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Abstract. A mathematical model for determining the amount of information applied to non-recoverable elements of the technical system is proposed. The quantitative indicators of the model are the probability of failure-free operation; frequency of failure; failure rate and mean time to the first failure. One of the criteria for assessing the reliability of the system adopted information entropy, which is considered as a measure of uncertainty of the outcome of a random event. The model allows to express the reliability indicators through information entropy for the analysis of the state of elements and the system. Multiplicative and additive properties of entropy growth in the case of aging and testing of system elements for resistance to failures are considered. An integral part of the model is a method for calculating the information entropy of systems consisting of non-recoverable elements. The method is based on the Shannon's model to determine the amount of information and allows to calculate the entropy for both operable and non-operable states of the system. The peculiarity of the method is as follows: 1) the probabilities of the operable state and the probability of failure in the case of considering the flow process of system states without taking into account the individual properties of the element are used to determine the information entropy. In this case, the result of the calculation allows to monitor the manifestation of randomness in the opposite states of the system; 2) it uses the individual properties of the element characterized by private (or own) entropy. With this approach to the calculations, the obtained quantitative values of entropy allow to monitor the growth rate of the states of each element and system as a whole. The method is tested on the presented example.

1. Introduction

The technical system is complex and requires analysis of the state of its elements. The complexity is due to the presence of not only a large number of elements, but also a branched structure of the connection between them. The elements function both independently and in mutual connection. Considering connections, or lack thereof, we can talk about stochastic processes. From the standpoint of reliability, the analysis of these independently functioning processes involves the use of probabilistic estimates and conditional distributions of indicators. At the same time, information of a statistical nature is often used. This information can be used to quantify the reliability of different devices.

There are several quantitative indicators that characterize the reliability of non-recoverable objects, among which are: probability of failure-free operation $p(t)$; frequency of failures $f(t)$ or $a(t)$; failure rate $\lambda(t)$; mean time to the first failure (MTTF) T_{mean} . In particular, the probability of failure-free



operation is understood as the probability that no failure will occur during operation in a given time interval. However, these data should be subject to analysis of the statistical distribution of random variables. The absence of this process doesn't guarantee an adequate result that would allow to assess the impact of undesirable factors on the state and reliability of the system.

2. The applicability of the criterion of information uncertainty measure

Entropy H , which is a measure of uncertainty of the outcome of a random experience or event [1], is considered as another criterion for assessing the reliability of the system.

With regard to the analysis of the reliability of technical systems, entropy is endowed with a number of properties [2]: 1) it is positive in magnitude; 2) it is equal to zero if the probability of occurrence of one of the events is equal to one; 3) it is maximal if all events are equally probable; 4) it is divided into components according to a qualitative criterion if there are opposing states (for example, operable and non-operable state of the element); 5) it has the property of additivity (when the state of the elements are considered as independent, the entropy of any complex system is determined as the sum of the private entropies of the elements).

Recall that the property of additivity in the mathematical formulation of the problem is generally expressed as:

$$y = \sum_{i=1}^n x_i. \quad (1)$$

In the expression (1) x_i is considered as information entropy.

The multiplicative property is also applicable. This property is represented by the formula:

$$y = \prod_{i=1}^n x_i. \quad (2)$$

The formula (2) is applicable, for example, for probabilities (in cases when the conditions of the multiplication theorem are satisfied).

If we analyse the reliability of systems from the standpoint of the presence of the amount of entropy, the property (1) will reflect the establishment of the connection between such effective indicators as the entropy of elements i [3]. Let's explain in some detail the role of entropy additivity in assessing the reliability of independent elements of the system. From the point of view of information theory, additive growth means the growth of entropy, the value of which depends on the number of n independent elements. The larger the system, the more information it carries. If we talk about the reliability of the system, then we can say that entropy is a measure of chaos in the system. It's about the fact that the system can be in two states: operable and non-operable.

Since chaos is not allowed in the technical system, the growth of entropy is not seen as an increase in chaos, but as an increase in orderliness associated with the operable state. To determine the amount of entropy of non-recoverable elements of the system, additive growth is associated with a change in the number of elements, an increase in the states of the element or the number of independent events. Multiplicative growth is directly related to the flow process of events related to the considered element or system. Thus, the amount of entropy reflects the influence of growth / decrease in the number of events and elements of the system necessary to assess the state of reliability.

Let's further propose the possibility of applying quantitative entropy values in the analysis of the reliability of the system in the presence of non-recoverable elements in it.

3. Entropy calculation method

The entropy calculation method should provide for the distribution of reliability parameters over time. Therefore, it is necessary not only to collect statistical data on the reliability of the system, but also to answer the question of whether discrete random variables belong to known distribution laws. The random nature of the parameter variation is very diverse, but in the reliability theory, for example, such laws of equipment failure distribution as exponential, normal and the Weibull distribution [4] are distinguished. In turn, the method of calculating entropy is associated with determining the type of

distribution law, that is, the selection of the analytical function that best approximates the empirical reliability functions. The selection procedure is largely subjective and depends on a priori knowledge about the system, its properties and structural connections. Therefore, the choice of distribution doesn't exclude from consideration the acceptance of the hypothesis, that the random variable $p(t)$, $f(t)$ or $\lambda(t)$ has a distribution of the proposed type [4, 5]. Since these indicators are associated with failures of the system elements, the type of distribution law will depend on the nature of the failures: gradual and sudden failures, which will be discussed further.

Calculation of the entropy of complex systems with non-recoverable elements assumes the following conditions: the flow of system failures is the simplest, stationary and ordinary. In this case, the elements of a complex system will be endowed with random and independent events. This statement and the chosen distribution law open the possibility of using the probability values of events or states for calculating entropy (according to the Shannon's model [6]). The obtained entropy values don't allow us to have a sufficient picture of the structural state and structural reliability of the system, since it is necessary to take into account the type of connections in the structure (that is, methods of connecting elements) and the nature of failures. Due to these circumstances, let's consider the sudden failures of the elements. The probability of failure-free operation of these elements can be determined (according to the above conditions) by the expression:

$$P_c = p_1 \cdot p_2 \cdot \dots \cdot p_n = \prod_{i=1}^n p_i, \quad (3)$$

where p_i is the probability of failure-free operation of the i -th element, n is the number of elements of the system.

The function (3) reflects the total probability of the flow process of independent events in time, provided it is stationary.

To analyse the role of entropy, it is worthwhile to consider some mathematical methods for calculating the amount of information. Let's turn to the Shannon's model for measuring the amount of information for states/events with different probabilities [6]. The connection between the probability of states and the amount of information (obtained at the occurrence of an event) is expressed by the Shannon's formula:

$$I = -\sum_{i=1}^N p_i \log_2 p_i, \text{ by } \sum_{i=1}^N p_i = 1, \quad (4)$$

where I is the amount of information; N is the number of possible events; p_i is the probability of the i -th event.

The expression (4) is true in cases where events appear independently of each other, so it is necessary to accept the condition: all probabilities are equal to the 1. The amount of information in the Shannon's formula is an average characteristic – the mathematical expectation of the distribution of a random variable $\{I_1, I_2, \dots, I_N\}$ and refers to the additive growth process.

According to the formula (4), it is possible to reflect the possibility of distinguishing information by a qualitative feature [7]:

$$I = -(\sum_{i=1}^{N_1} p_i \log_2 p_i + \sum_{j=1}^{N_0} q_j \log_2 q_j), \text{ by } \sum_{i=1}^{N_1} p_i + \sum_{j=1}^{N_0} q_j = 1, \quad (5)$$

where p_i and q_j are the probabilities of opposite states (for example, p_i is the probability of an operable state, q_j is the probability of a non-operable state), N_1 and N_0 are the number of events related to the probability of maintaining an operable state and the probability of a non-operable state.

The need to distinguish information on each of the states is explained by the requirements for reliability, when its analysis highlights the opposite state: operable and non-operable.

The expressions (4) and (5) are also applicable to determine the information entropy H , since it carries elements of randomness, that is, the probability of occurrence of events. The probability of the event in (4), by definition, differs from the probability of failure-free operation in (3). They don't agree with each other for the following reason: the probability of failure-free operation of the i -th

element is considered in the selected time interval, whereas the probability of the i -th event is an individual value related to a particular state of the element. To exclude this kind of disagreement, we will resort to the following arguments. Let the entropy calculations be based on the number of states of the system. Each element of the system can be in one of the two states (1 – operable; 0 – non-operable), then the system (from the position of equally probable outcomes) will have $N=2^n$ states. The complete combination of N equally probable states provides the maximum amount of information about the state of the entire system. In this case, the Shannon's formula (1) is simplified and transformed into the R. Hartley's formula [8] for equally probable states:

$$H = \sum_{i=1}^N \frac{1}{N} \log_2 \frac{N}{1} = N \frac{1}{N} \log_2 N = \log_2 N. \quad (6)$$

Based on the formula (6), let's substitute 2^n instead of N , where n is the number of elements in the system. After minor transformations, taking into account the qualitative division of entropy in (5) into components, the expression (6) is represented as:

$$H = -n[p(t) \log_2 p(t) + q(t) \log_2 q(t)]. \quad (7)$$

Expressions (4)-(7) refer to the possibility of applying a syntactic measure of information when the amount of information reflects impersonal information that doesn't express the semantic content of the object. Next, the example shows the essence of the amount of information obtained from these mathematical expressions.

The content of the element (with some share of the object properties) will be reflected through the private information, which was initially assigned to the object after assessing the degree and significance (for example, in terms of reliability) of its characteristics. For example, in the case of non-recoverable elements, after their manufacture, they are tested to determine the frequency of failures $f(t)$, failure rate $\lambda(t)$ and mean time to the first failure of T_{mean} . The value of these parameters allows to determine the amount of private information about the object. The word "private" means that the information belongs to only one object and it is obtained based on probabilistic or frequency characteristics and is taken constant, based on the condition of the invariance of the object properties in the considered time interval. Here is an example of its calculation: when testing the reliability of the object, the average failure rate is 1 failure per 100 tests, then, the probability of failure-free operation is equal to 0.99, and the probability of failure is equal to 0.01.

Private information (or entropy) is calculated according to the expression (6), in which both the number of states (according to Hartley's approach) and the probabilities (according to Shannon's approach) can be present:

$$h = \log_2 p. \quad (8)$$

Taking into account this entropy, expression (7) is transformed and will look like:

$$H = -n[p(t) \log_2 p + q(t) \log_2 q]. \quad (9)$$

The expression (9) is valid for determining the entropy of the system state in the considered time interval Δt , and the quantitative criteria for determining H (according to the classical reliability theory) are determined by the formulas:

$$\text{- probability of failure-free operation, } p(t) = \lim_{\substack{\Delta t \rightarrow 0 \\ N_0 \rightarrow \infty}} \frac{N_0 - \sum_{i=1}^{t/\Delta t} n_i}{N_0} \approx \frac{N(t)}{N_0},$$

$$\text{- probability of failure, } q(t) = \lim_{\substack{\Delta t \rightarrow 0 \\ N_0 \rightarrow \infty}} \frac{\sum_{i=1}^{t/\Delta t} n_i}{N_0},$$

where N_0 is the number of elements at the beginning of the system tests; n_i is the number of failed elements in the time interval Δt ; t is the time for which the probability is determined; $N(t)$ is the number of elements working properly in the interval $[0; t]$.

4. Example of solving the problem

The same type of products, after conducting the initial failure tests, has a private entropy of the operable state $-\log_2 p = \log_2 0.92$ and the non-operable state $-\log_2 q = \log_2 0.08$. For carrying out repeated check on reliability, a batch of 500 similar products is taken. The tests were carried out for 5000 hours. During the first 3000 hours, 40 items were rejected, 25 items were rejected in the following 2000 hours. It is required to determine the entropy components at intervals $[0; 3000] \rightarrow \Delta_1$ and $[3000; 5000] \rightarrow \Delta_2$ hours and to compare them with each other to determine the degree of reliability.

Decision. Pre-calculate the probability in the considered time intervals:

- probability of failure-free operation in the interval Δ_1 :

$$p(\Delta_1) \approx \frac{N(\Delta_1)}{N_0} = \frac{460}{500} = 0.92, \text{ where } N(\Delta_1) = N_0 - \sum n_i = 500 - 40 = 460;$$

- probability of failure-free operation in the interval Δ_2 :

$$p(\Delta_2) \approx \frac{N(\Delta_2)}{N_0} = \frac{435}{500} = 0.87, \text{ where } N(\Delta_2) = N_0 - \sum n_i = 500 - (40 + 25) = 435;$$

$$\text{- probability of failure in the interval } \Delta_1: q(\Delta_1) = \frac{\sum n_i}{N_0} = \frac{40}{500} = 0.08,$$

$$\text{- probability of failure in the interval } \Delta_2: q(\Delta_2) = \frac{\sum n_i}{N_0} = \frac{40 + 25}{500} = 0.13.$$

For each considered interval, the following condition is met: $p(\Delta_i) + q(\Delta_i) = 1$. Next, we apply expression (7) without taking into account the private entropy.

Entropy in the interval Δ_1 :

$$H(\Delta_1) = -N_0 [p(\Delta_1) \log_2 p(\Delta_1) + q(\Delta_1) \log_2 q(\Delta_1)] = H[p(\Delta_1)] + H[q(\Delta_1)] = \quad (10)$$

$$= -500 \cdot (0.92 \log_2 0.92 + 0.08 \log_2 0.08) = 55.34 + 145.75 = 201.09 \text{ bit}.$$

Entropy in the interval Δ_2 :

$$H(\Delta_2) = -N_0 [p(\Delta_2) \log_2 p(\Delta_2) + q(\Delta_2) \log_2 q(\Delta_2)] = \quad (11)$$

$$= -500 \cdot (0.87 \log_2 0.87 + 0.13 \log_2 0.13) = 87.40 + 191.32 = 278.72 \text{ bit}.$$

Comparing the quantitative values of the entropy obtained according to (10) and (11) in the considered time intervals, the following can be seen: $H(\Delta_2) > H(\Delta_1)$, as a consequence of the increase in the number of failed products in time, entropy increases and tends to the maximum with the equality of opposite probabilities:

$$H_{\max} = -500 \cdot (0.5 \log_2 0.5 + 0.5 \log_2 0.5) = 250 + 250 = 500 \text{ bit}.$$

Now we apply expression (9) taking into account the private entropy. The calculation of $H(\Delta_1)$ according to the expression (10) remains unchanged, since the probabilities of primary and subsequent tests in the interval $[0; 3000]$ coincide.

Entropy in the interval Δ_2 :

$$H(\Delta_2) = -N_0 [p(\Delta_2) \log_2 p(\Delta_2) + q(\Delta_2) \log_2 q(\Delta_2)] = H[p(\Delta_2)] + H[q(\Delta_2)] = \quad (12)$$

$$= -500 \cdot (0.87 \log_2 0.92 + 0.13 \log_2 0.08) = 52.33 + 236.85 = 289.18 \text{ bit}.$$

Comparing the values of entropy obtained according to (10) and (12), let's note the following: the increase in the number of failures, on the one hand, indicates a decrease in the probability of failure-free operation $p(\Delta_2) < q(\Delta_1)$ and a decrease in entropy $H[p(\Delta_2)] < H[p(\Delta_1)]$, on the other – increases the probability of failure and leads to an increase in entropy $H[q(\Delta_2)] > H[q(\Delta_1)]$. Thus, a

decrease in the level of reliability of the tested products is associated with an increase in the entropy of the non-operable state and a decrease in the operable state.

The peculiarity of applying formula (9) is that its private entropy about the state of the object takes part in obtaining a posteriori information in the case of changes in its states over time.

According to (9), in the process of "aging" of the system or increasing the number of failed elements in additional tests, the total increase of entropy is due to the growth of the entropy of failure. An increase in the values of this entropy indicates an increase in the number of various failure states, whereas for highly reliable systems (with a small amount of private entropy) the entropy of the failure-free state decreases, indicating a decrease in the states of failure-free operation of the system elements.

5. Conclusion

Entropy as a measure of information uncertainty can be used as a quantitative criterion for assessing the level of reliability of a system consisting of non-recoverable elements. To determine it, it is necessary to observe a number of conditions related both to the regularities of the distribution of random variables, and to the properties inherent in entropy. The method of entropy determination is simple and is based on Shannon's mathematical model of the logarithmic distribution of random events. An important feature of the method is to take into account the private entropy of the system element during its reliability test. This approach to the calculation of entropy is fundamentally different from its classical definition according to Shannon. With the help of the proposed method, it is possible to determine not only the entropy of the system state as a whole, but also to calculate its components: the entropy of the operable and non-operable states of the system. These components make it possible to judge the reliability of the system in conjunction with the information uncertainty about its state.

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